J. Heat Mass Transfer. Vol. 17, pp. 1063 1077. Pergamon Press 1974. Printed in Great Britain

EXTINCTION ANALYSIS OF PREMIXED FLAME FOR COUNTER FLOW AND BLUNT BODY FORWARD STAGNATION REGION FLOW

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(Received 17 October 1973)

Abstract—An extinction analysis was carried out for the counter flow and blunt body forward stagnation region premixed flames under two-dimensional and axially symmetrical viscous compressible laminar boundary layer approximations.

A new non-dimensional similarity parameter defined as maximum local Damköhler number was adopted as an extinction criterion of premixed flames. The effects of varying the concentrations of fuel and oxidant, free stream temperature, activation energy, etc. upon this similarity parameter were examined for both flow geometries. It was concluded from present analysis that "extinction" of flames takes place at nearly constant values of the maximum Damköhler number.

NOMENCLATURE

- a, characteristic time of flow [1/s];
- A, $= \rho\mu/\rho_e\mu_e$;
- $\bar{C}_f$, $\bar{C}_{\bar{o}}$, mass fraction of fuel and oxidant, respectively;
- C_i, normalized mass fraction of species i;
- C_p, specific heat at constant pressure [cal/g deg K];
- D, local Damköhler number;
- D_1, first Damköhler number $= \nu_1^o \nu_1^f \rho_e K/(k+1)am$;
- D_m, maximum local Damköhler number $= \{D_1 C_o C_f \exp(-\theta/T)\}_{\max}$;
- D_{12}, binary diffusion coefficient of component 1 and component 2 [cm^2/s];
- D_i, diffusion coefficient of species i [cm^2/s];
- E, activation energy [cal/mole];
- f, $= \psi(x, y)/\sqrt{(2s)}$; $\sqrt{(2s)} = \sqrt{\left(\frac{\rho_e\mu_e a}{k+1}\right)}x^{k+1}$ for two-dimensional and axisymmetrical stagnation flow;
- h_i, specific enthalpy [cal/g];
- h_i^0, heat of formation at temperature T_0[cal/g];
- Δh_c, heat of combustion of fuel [cal/g fuel];
- ΔH_c, heat of combustion of fuel [cal/g mole fuel];
- k, constant = 0 for two-dimensional flow and = 1 for axially symmetrical flow;
- K, frequency factor [cm^3/mole s];
- Le_i, Lewis number of species $i = Sc_i/Pr$;
- m, $= \sum_i \nu_1^i m_i = \sum_i \nu_2^i m_i$;
- m_i, molecular weight of species i;
- p, pressure [atm];
- $\dot{q}$, heat flux in the y direction [cal/cm^2 s];
- Pr, Prandtl number;
- R, universal gas constant [1·986 cal/g mole deg K];
- r_0, radius of the curvature of the blunt body [cm];
- Sc_i, Schmidt number of species i;
- $\bar{T}$, temperature [deg K];
- T, non-dimensional temperature, $= C_p\alpha_f \bar{T}/\Delta h_c$;
- u, v, velocities in the directions of x and y [cm/s];
- V_i, diffusion velocity of species i [cm^2/s];
- $\dot{w}_i$, reaction rate of species i [moles/cm^2 s];
- x, y, distance from stagnation point along and perpendicular to stagnation plane or blunt body surface, respectively [cm].

Greek symbols

- α_i, $= -m/(\nu_2^i - \nu_1^i)m_i$;
- η, similarity variable, $= \frac{u_e r_0^k}{\sqrt{2s}}\int_0^y \rho\,dy$; $\eta = \sqrt{\left[\frac{(k+1)a}{\nu_e}\right]}\int_0^y \frac{\rho}{\rho_e}dy$ for plane and axisymmetrical stagnation flow;
- θ, activation temperature $= EC_p\alpha_f/R\,.\,\Delta h_c$;
- λ, thermal conductivity [cal/cm s deg K];
- μ, viscosity [g/cm s];
- ν, kinematic viscosity [cm^2/s];
- ν_1^i, ν_2^j, stoichiometric coefficients of the reactant i and the product j, respectively;
- ρ, density [g/cm^3];
- ψ, stream function $= \sqrt{(2s)}f$.

Subscripts and superscripts

e,	edge of boundary layer;
ext,	the value at extinction;
f,	fuel;
m, max,	maximum value;
$\bar{o}$,	oxidant.

1. INTRODUCTION

IN THE last two decades, considerable amounts of theoretical investigations have reported for the chemically reacting laminar boundary-layer flows of both initially unmixed and premixed gases.

Among them interesting features of extinction and ignition problems have been revealed in forced convection systems of counter flow stagnation, blunt body stagnation and wall jet configurations. The problems of extinction and ignition of flames in such systems are of practical importance in transpilation cooling, ablation cooling in re-entry vehicles and flame stabilization in high speed flow combustion chamber. Spalding [1] has been the first to treat the problem of counter flow diffusion flame theoretically and predict the condition under which the flame would extinguish with increasing mass flux. Later, Anagnostou and Potter [2] verified Spalding's work experimentally.

A thoroughful analysis has been rendered by Fendell [3], in which the extinction and ignition of flame stabilized in the vicinity of an axisymmetric stagnation point are treated, when jet of oxidant oncomes from an upstream infinity toward a reservoir of liquid fuel. Inner and outer expansion techniques for singular perturbation method were adopted to obtain the chemically frozen and equilibrium states. He was the first to surmise that the first Damköhler number which is the ratio of the time characterizing flow to the time characterizing chemical reaction has important role in problems of extinction and ignition. From the fact that in almost all works on extinction and ignition since Fendell, this similarity parameter has used to account for the phenomena, Fendell's contribution in this field is of significance.

Tsuji and Yamaoka [4] have performed the experiment on extinction of wall jet diffusion flame near stagnation point of porous cylinder through which fuels were injected. Theoretical analysis was also performed to adopt Burke–Schumann thin flame kinetics, although no prediction of extinction limit has obtained.

Mori *et al.* [5] have investigated on the axisymmetric counter flow diffusion flame and proposed to use Damköhler number which contains maximum temperature.

Jain and Mukunda [6] has analyzed the problem of extinction and ignition of the wall jet and counter flow diffusion flames under the three approximations in momentum equation, namely potential flow, viscous incompressible flow and Lees' approximation, adopting Spalding's independent variable to reduce the range of numerical integration. They have extended their analysis involving the competitive reactions [7].

Saitoh [8] has performed the extinction analysis of counter flow diffusion flame for inviscid incompressible flow, in which a new similarity parameter "maximum local Damköhler number" for extinction criterion has been proposed to prescribe the extinction conditions. The effects of varying the concentrations of fuel and oxidant, activation energy, etc. upon the critical value of this parameter were examined and it was verified that this similarity group at extinction maintains almost constant value in despite of wide change of these conditions.

While, extinction problem of blunt body forward stagnation region premixed flame was studied by Chambré [9] and later by Sharma and Sirignano [10]. They took up same problem under more realistic formulations. The effects of Lewis number were considered in their analysis but the properties were estimated at frozen state.

Takeno [11, 12] also has analyzed the above problem and suggested to define the burning velocity of the flame at reflection point of the temperature distribution.

Recently, Alkidas and Durbetaki [13] have considered the heat interaction between a combustible mixture and a constant temperature surface near the stagnation region of a blunt body and showed that surface heat transfer is critically affected by the first Damköhler number.

In present paper, the extinction analysis was carried out for counter flow stagnation region and blunt body forward stagnation region premixed flames under viscous compressible laminar boundary-layer approximations with two-dimensional and axisymmetric geometries. From the view point that "flame extinction" would occur at maximum reaction rate region, where maximum strength of flame could be expected, local similarity parameter was proposed as an extinction criterion of both premixed and diffusion flames.

It was shown that variations of the concentrations of fuel and oxidant, activation energy, jet temperature, heat of combustion of fuel and kinds of fuel have little effects upon the maximum local Damköhler number at extinction.

Consequently, the flame can be considered to extinguish at nearly the constant values of this similarity parameter.

Furthermore, the effects of flow patterns on the criterion was examined for counter flow and blunt body stagnation flow and also the effect of Lewis number was made clear by taking the properties at high temperature.

2. GOVERNING EQUATIONS

Descriptions of steady laminar boundary-layer flow over a planar body or an unyawed body of revolution for a chemically reacting mixture are as follows [10].

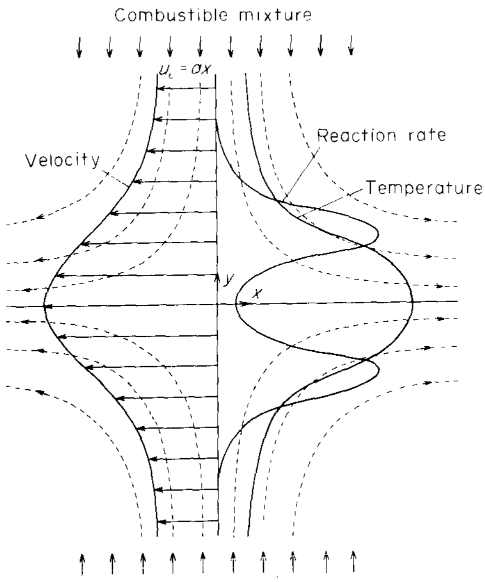

FIG. 1. Schematic diagram and co-ordinate system for counter flow premixed flame.

The co-ordinate systems are illustrated in Fig. 1 for counter flow and in Fig. 2 for blunt body forward stagnation flow geometries.

Continuity equation

$$\frac{\partial(\rho u r_0^k)}{\partial x}+\frac{\partial(\rho v r_0^k)}{\partial y}=0. \tag{1}$$

Momentum equation

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right). \tag{2}$$

Energy equation

$$\rho C_p u\frac{\partial \bar{T}}{\partial x}+\rho C_p v\frac{\partial \bar{T}}{\partial y}=\frac{\partial}{\partial y}\left(\lambda\frac{\partial \bar{T}}{\partial y}\right)+u\frac{\mathrm{d}p}{\mathrm{d}x}+\mu\left(\frac{\partial u}{\partial y}\right)^2$$
$$-\frac{\partial \dot{q}}{\partial y}+\sum_i h_i\frac{\partial}{\partial y}(\rho\bar{C}_iV_i)-\sum_i \dot{w}_ih_i(v_2^i-v_1^i)m_i. \tag{3}$$

Species conservation equation

$$\rho u\frac{\partial \bar{C}_i}{\partial x}+\rho v\frac{\partial \bar{C}_i}{\partial y}=\frac{\partial}{\partial y}(\rho\bar{C}_iV_i)+\dot{w}_i(v_2^i-v_1^i)m_i. \tag{4}$$

Equation of state

$$p=\rho R\bar{T}\sum_i\frac{\bar{C}_i}{m_i}. \tag{5}$$

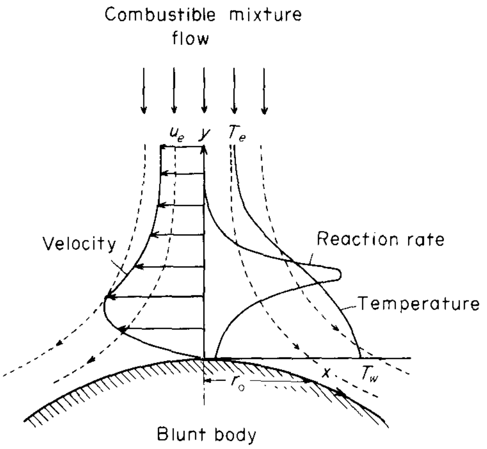

FIG. 2. Schematic diagram and co-ordinate system for blunt body stagnation region premixed flame.

Here, h_i denotes the static specific enthalpy of species i and is expressed as

$$h_i=\int_{T_0}^{\bar{T}}C_{p,i}\,\mathrm{d}\bar{T}+h_i^0. \tag{6}$$

In the analysis, the following assumptions are made.

(1) Flow is steady laminar and Eckert number is much smaller than unity, i.e. moderate speed flow.
(2) Boundary-layer approximation is valid.
(3) Specific heat at constant pressure of all species is constant and same, namely $C_{p,i}=C_p$.
(4) Dufour–Soret effect is negligible.
(5) Radiation effect is minor.
(6) Mixture consists of four components; fuel, oxygen, product and inert gas.
(7) Reaction is single step and a second order Arrhenius kinetics is valid.

Equation (5) is simplified by the mean molecular weight of free stream composition.

$$p=\rho R\bar{T}/\bar{M}. \tag{7}$$

From the above assumption (4) and assuming that the diffusion of reactants obeys Fick's law, the diffusion velocity is expressed by

$$\bar{C}_iV_i\cong-D_{12}\frac{\partial \bar{C}_i}{\partial y}. \tag{8}$$

Applying Bernoulli's law for the pressure distribution in the boundary layer, one obtains,

$$-\frac{\mathrm{d}p}{\mathrm{d}x}=\rho_eu_e\frac{\mathrm{d}u_e}{\mathrm{d}x}. \tag{9}$$

The rate of heat energy transported across the stream line for our flow configurations is represented by

$$\dot{q}=-\lambda\frac{\partial \bar{T}}{\partial y}+\sum_i\rho\bar{C}_iV_ih_i^0. \tag{10}$$

Using earlier mentioned assumptions and equations (8), (9), (10), basic equations (2)–(4) can be rewritten as follows.

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial v}{\partial y}=\rho_e u_e\frac{\mathrm{d}u_e}{\mathrm{d}x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (11)$$

$$\rho C_p u\frac{\partial \bar{T}}{\partial x}+\rho C_p v\frac{\partial \bar{T}}{\partial y}=\frac{\partial}{\partial y}\left(\lambda\frac{\partial \bar{T}}{\partial y}\right)+\Delta H_c\cdot\dot{w} \quad (12)$$

$$\rho u\frac{\partial \bar{C}_i}{\partial x}+\rho v\frac{\partial \bar{C}_i}{\partial y}=\frac{\partial}{\partial y}\left(\rho D_i\frac{\partial \bar{C}_i}{\partial y}\right)+(v_2^i-v_1^i)m_i\dot{w}. \quad (13)$$

Boundary conditions for velocity, temperature and concentration are;

Velocity;

(a) counter flow:

$$\frac{\partial u(x,0)}{\partial y}=0,\quad v(x,0)=0,\quad u(x,\infty)=u_e.$$

(b) blunt body stagnation flow:

$$u(x,0)=v(x,0)=0,\quad u(x,\infty)=u_e. \quad (14)$$

Temperature (for both flows);

$$\frac{\partial \bar{T}(x,0)}{\partial y}=0,\quad \bar{T}(x,\infty)=\bar{T}_e. \quad (15)$$

Concentration (for both flows);

$$\frac{\partial \bar{C}_i(x,0)}{\partial y}=0,\quad \bar{C}_i(x,\infty)=\bar{C}_{ie}. \quad (16)$$

To obtain the equation in a form compatible with similar solutions, the customary Mangler and Howarth–Dorodnitsyn transformation [14] is introduced.

$$s=\int_0^x \rho_e\mu_e u_e r_0^{2k}\,\mathrm{d}x, \quad (17)$$

$$\eta=\frac{u_e r_0^k}{\sqrt{2s}}\int_0^y \rho\,\mathrm{d}y. \quad (18)$$

The stream function ψ is defined to satisfy the equation of continuity (1).

$$\rho u r_0^k=\frac{\partial\psi}{\partial y},\quad \rho v r_0^k=-\frac{\partial\psi}{\partial x}. \quad (19)$$

The function f is defined by the following equation to transform the momentum equation.

$$f(s,\eta)=\frac{\psi(x,y)}{\sqrt{2s}}. \quad (20)$$

In virtue of f, the velocity components are expressed as

$$\frac{u}{u_e}=\frac{\partial f}{\partial\eta}=f', \quad (21)$$

$$\rho v=-\frac{\rho_e\mu_e u_e r_0^k}{\sqrt{2s}}\left\{f+\frac{2s}{\rho_e\mu_e u_e r_0^{2k}}\frac{\partial\eta}{\partial x}\frac{\partial f}{\partial\eta}+2s\frac{\partial f}{\partial\eta}\right\}. \quad (22)$$

Before describing the full equations by the above variables, we confine the problem to two-dimensional and axisymmetrical stagnation flow. Therefore, the governing equations can be transformed to the following equations with f, $\bar{T}$ and $\bar{C}_i$ regarded as the function of only η.

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(A\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2}\right)+f\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2}+\frac{1}{k+1}\left[\frac{\rho_e}{\rho}-\left(\frac{\mathrm{d}f}{\mathrm{d}\eta}\right)^2\right]=0 \quad (23)$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{A}{Pr}\frac{\mathrm{d}\bar{T}}{\mathrm{d}\eta}\right)+f\frac{\mathrm{d}\bar{T}}{\mathrm{d}\eta}=-\frac{\Delta H_c\dot{w}}{\rho a(k+1)C_p} \quad (24)$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{A}{Sc_i}\frac{\mathrm{d}\bar{C}_i}{\mathrm{d}\eta}\right)+f\frac{\mathrm{d}\bar{C}_i}{\mathrm{d}\eta}=-\frac{(v_2^i-v_1^i)m_i\dot{w}}{\rho a(k+1)}. \quad (25)$$

Where,

$$\eta=\sqrt{\left(\frac{(k+1)a}{v_e}\right)}\int_0^y\frac{\rho}{\rho_e}\mathrm{d}y \quad (26)$$

and

$$A=\frac{\rho\mu}{\rho_e\mu_e}. \quad (27)$$

It may be assumed that the reaction rate is expressed by the Arrhenius second order equation, that is,

$$\dot{w}=\frac{K\rho^2\bar{C}_o\bar{C}_f}{m_o m_f}\exp\left(-\frac{E}{R\bar{T}}\right). \quad (28)$$

Without compromising fundamental physics, the approximations (i) $A=\rho\mu/\rho_e\mu_e=1$ and (ii) Pr and Sc_i are constant are adopted for simplification. These approximations are found to be generally good for many situations.

Further approximation of Lewis number unity is made in the analysis except a few general cases, when the more realistic values of properties estimated at high temperature are used to verify the validity of an assumption of Lewis number unity.

Non-dimensional parameters used by Fendell are adopted.

$$T=\frac{mC_p\bar{T}}{\Delta H_c}=\frac{C_p\alpha_f}{\Delta h_c}\bar{T}, \quad (29)$$

$$C_i=\alpha_i\bar{C}_i. \quad (30)$$

here,

$$\alpha_i=-\frac{m}{(v_2^i-v_1^i)m_i}. \quad (31)$$

$$m=\sum_i v_1^i m_i=\sum_i v_2^i m_i. \quad (32)$$

Taking account of above mentioned simplifications, the final set of the governing equations may be written as follows.

$$f'''+ff''+\frac{1}{k+1}\left[\frac{T}{T_e}-(f')^2\right]=0 \quad (33)$$

$$T''+PrfT'=-D_1PrC_oC_f\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T} \quad (34)$$

$$C_i''+PrLe_ifC_i'=D_1PrLe_iC_oC_f\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T} \quad (35)$$

Where,

$$\theta = \frac{E}{R}\frac{C_p}{\Delta h_c}.\alpha_f, \tag{36}$$

$$D_1 = \frac{\nu_1^{\bar{o}}\nu_1^{f}\rho_e K}{(k+1)am}, \tag{37}$$

$$Le_i = \frac{Sc_i}{Pr}. \tag{38}$$

The primes denote differentiation with respect to η and D_1 is the first Damköhler number similarity parameter first introduced by Fendell.

Boundary conditions are listed as follows.

Velocity;

(a) counter flow;

$$f(0) = f''(0) = 0, \quad f'(\infty) = 1. \tag{39}$$

(b) blunt body stagnation flow;

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1. \tag{40}$$

Temperature;

$$T'(0) = 0, \quad T(\infty) = T_e. \tag{41}$$

Concentration;

$$C_i'(0) = 0, \quad C_i(\infty) = C_{ie}. \tag{42}$$

A greater simplification of $Le_i = 1$ which is first used by Schwav and Zeldovitch [15] is made. Four ordinary differential equations reduce two by the use of this relation, that is, equation (34) is expressed as

$$T'' + PrfT' = -D_1(T_e + C_{fe} - T) \times (T_e + C_{\bar{o}e} - T)\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T}. \tag{43}$$

Then, the problem reduces to find the eigen value D_1 under given values of T_e, C_{fe}, $C_{\bar{o}e}$, θ, Pr and $T(0)$.

3. NUMERICAL PROCEDURE

Numerical integrations of the momentum equation (33) and the energy equation (43) were performed by Gill's modified Runge-Kutta fourth order method. First, equation (43) was integrated for given data T_e, $C_{\bar{o}e}$, C_{fe}, θ, Pr and $T(0)$ with assumed initial value of D_1, with f being set to $f = \eta$.

Iteration was continued until the boundary condition $T(\infty) = T_e$ was satisfied, and then the momentum equation (33) was integrated with assumed initial value of $f'(0)$ for counter flow and of $f''(0)$ for blunt body stagnation flow.

Bessel's interpolation formula was used to estimate the value of T or f at half way of interval.

Similarly, successive iterations were continued until the sufficient accuracy was reached.

An automatic Fortran program for above numerical calculation was made for the sake of great reduction of computing work.

4. RESULTS AND DISCUSSION

Numerical calculations were carried out for the mixture of ethylene–air and propane–air with various mixing ratio.

Same composition mixture impinges with same velocity in case of counter flow premixed flame.

Following physical values were used in the calculations [15, 16].

For C_2H_4;

$T_e = 300°K$, $C_p = 0{\cdot}3$ cal/g°K, $\Delta h_c = 11\,270$ cal/g, $E = 20\,000$ cal/mole, $Pr = 0{\cdot}75$, $Le_i = 1$.

For C_3H_8;

$T_e = 300°K$, $C_p = 0{\cdot}32$ cal/g°K, $\Delta h_c = 11\,120$ cal/g, $E = 26\,000$ cal/mole, $Pr = 0{\cdot}75$, $Le_i = 1$.

4.1 *Typical structure of flames*

Figure 3 shows the typical structure of counter flow premixed flame for ethylene–air stoichiometric mixture.

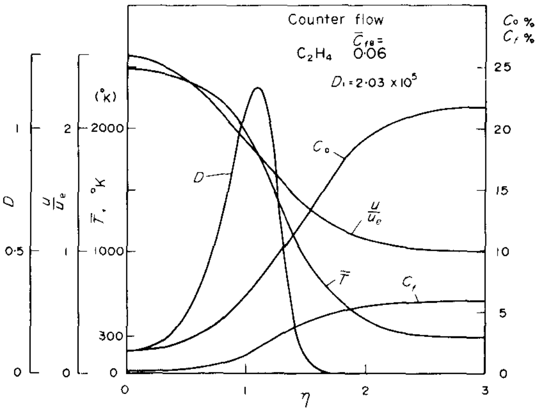

FIG. 3. Typical structure of counter flow premixed flame.

Those shown in the figure are the distributions of temperature, concentrations of fuel and oxygen, velocity in the direction of x and local Damköhler number defined as

$$D = D_1 C_{\bar{o}} C_f \exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T}. \tag{44}$$

All distributions are symmetrical with respect to stagnation plane and two reaction zones are formed except extinction state. Temperature distribution has its maximum at stagnation plane, on the other hand the distribution of local Damköhler number takes the maximum value at the position away from the stagnation when $T(0)$ is higher than extinction temperature, and at stagnation when $T(0)$ is lower than extinction temperature.

Typical distributions for blunt body stagnation flow are shown in Fig. 4. Difference of flow configuration is apparent from Figs. 3 and 4.

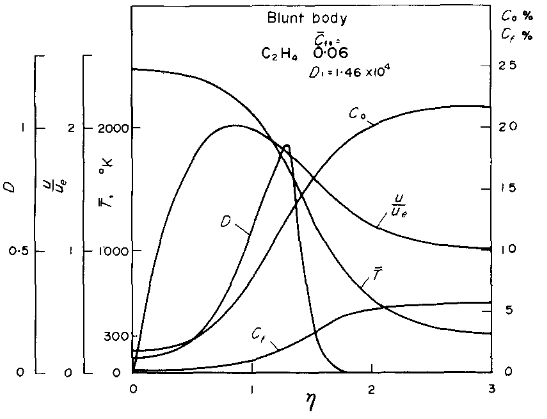

FIG. 4. Typical structure of blunt body stagnation region premixed flame.

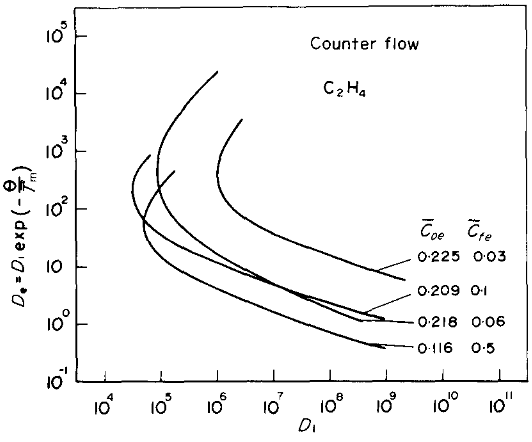

FIG. 5. Plot of Mori *et al.*'s Damköhler number D_e vs D_1 with varying concentrations for counter flow premixed flame.

4.2 *Maximum local Damköhler number and flame extinction*

Since it can be considered that extinction of flame occurs at a narrow region called reaction zone, a parameter which describes extinction should be defined by the "local value" at the reaction zone. It seems that no exact explanation for extinction phenomena can be given by the quantity at cold state.

Fendell [3] has been the first to recognize the first Damköhler number D_1 as a criterion of extinction by plotting the maximum temperature against D_1. However, the value of D_1 at extinction varies considerably against activation energy change. This fact shows that realistic descriptions are not available by the value at frozen state.

Mori *et al.* [6] have suggested to adopt the local value $D_e = D_1 \exp(-\theta/T_m)$ for another criterion of extinction, considering that the temperature dependence of extinction parameter may be strong.

It was noted that the effect of varying activation energy upon extinction value of D_e is comparatively minor to that upon $D_{1,\mathrm{ext}}$. Yet, as shown in Fig. 5, $D_{e,\mathrm{ext}}$ moves considerably with variation of fuel and oxidant concentrations. The cause of these scattering is due to the fact that the maximum temperature alone is considered as the local quantity of the reaction zone.

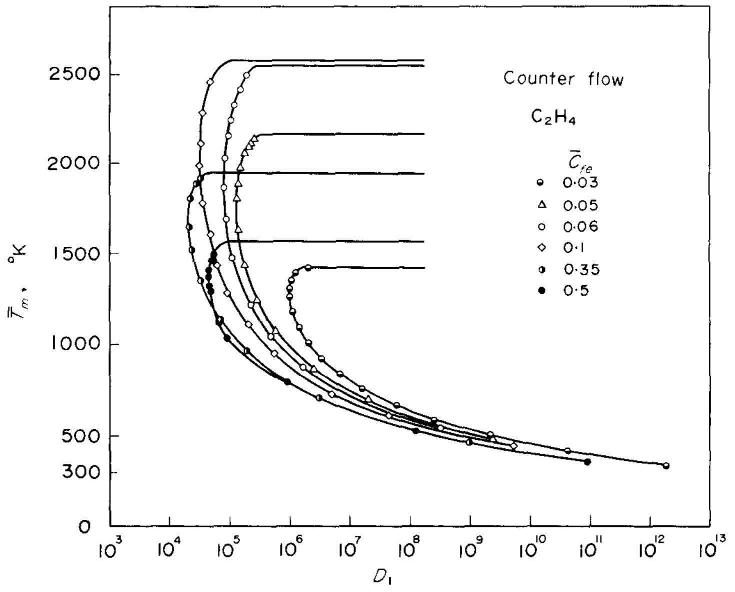

FIG. 6. Plot of maximum temperature T_m vs D_1 (Fendell curves) showing extinction states for counter flow premixed flame.

As mentioned earlier, the local quantity at maximum reaction rate point has a great influence on the extinction of flames, so the ratio of convection time to chemical time at the maximum reaction rate point, i.e. the maximum local Damköhler number point becomes most important parameter for describing extinction phenomena.

By the definition, this parameter can be written as

$$D_m = \left\{D_1 \,.\,(T_e + C_{fe} - T) \times (T_e + C_{\bar{o}e} - T)\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T}\right\}_{\max}. \quad (45)$$

Figure 6 shows the plots of maximum temperature T_m vs first Damköhler number (referred to Fendell curves) for ethylene–air premixed flame with various fuel and oxidant concentrations, which first considered by Fendell to predict the flame extinction.

While, the same results were rearranged by use of the maximum local Damköhler number in Fig. 7. Remarkable features can be found from this figure. The extinction points which are defined by the nose points in Fig. 6 lies almost at constant values of D_m in despite of the concentration variation of fuel and oxidant. Another noticeable point is that a fairly good similarity holds for in the middle and lower branch in Fendell curves.

From the fact illustrated by the figures, it can be stated that the flames extinguish at nearly constant value of the maximum local Damköhler number.

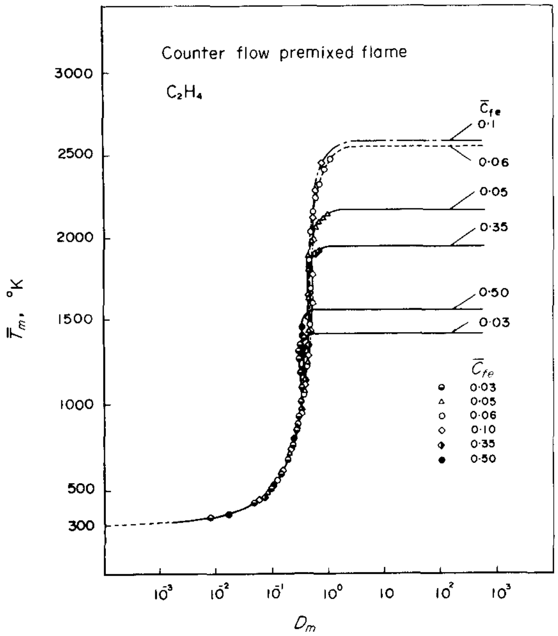

FIG. 7. Plot of maximum temperature T_m vs maximum local Damköhler number D_m corresponding to Fig. 6 for counter flow premixed flame.

4.3 *Extinction temperature and maximum local Damköhler number*

The local Damköhler number

$$D = D_1(T_e + C_{fe} - T)(T_e + C_{\bar{o}e} - T)\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T} \quad (46)$$

is a function of temperature T only for given values of D_1, T_e, C_{fe}, C_{oe} and θ. Therefore, its maximum takes place at the point where the following relation is satisfied.

$$\frac{\mathrm{d}D}{\mathrm{d}T} = 0 \quad (47)$$

Consequently, the flame temperature at maximum local Damköhler point, i.e. at maximum reaction rate point has always a constant value T_{ext} when the position of D_m is not placed at stagnation. Provided that $T(0)$ is less than T_{ext}, the positions of D_m are of necessity located at stagnation, i.e. $\eta = 0$.

As seen from the fact stated above, it can be considered that the extinction point is reflection point, and then a following relation holds for at extinction.

$$\frac{\mathrm{d}^2D}{\mathrm{d}T^2} = 0. \quad (48)$$

Substituting equation (46) into (47), one obtains the extinction temperature T_{ext} by solving the cubic equation written below.

$$T_{ext}^3 + \alpha T_{ext}^2 + \beta T_{ext} + \gamma = 0 \quad (49)$$

where

$$\left.\begin{aligned} \alpha &= -(C_{\bar{o}e} + C_{fe} - 3T_e - \theta)/2, \\ \beta &= -(C_{\bar{o}e} + C_{fe} - 2T_e)(\theta - T_e)/2, \\ \gamma &= (C_{\bar{o}e} + T_e)(C_{fe} + T_e)(\theta - T_e)/2. \end{aligned}\right\} \quad (50)$$

Meaningful root of the equation (49) is given by the formula.

$$T_{ext} = 2\sqrt{-p}\,\cos\left(\frac{u}{3} + \frac{2}{3}\pi\right) \quad (51)$$

here,

$$\begin{aligned} u &= \cos^{-1}\left\{\frac{q}{p\sqrt{-p}}\right\}, \\ p &= -\left(\frac{\alpha}{3}\right)^2 + \frac{\beta}{3}, \\ q &= \left(\frac{\alpha}{3}\right)^2 - \frac{\alpha\beta}{6} + \frac{\gamma}{2}. \end{aligned} \quad (52)$$

Then, the maximum Damköhler number can be calculated by following equations;

(i) in case of $T(0) > T_{ext}$

$$D_m = D_1(T_e + C_{fe} - T_{ext}) (T_e + C_{\bar{o}e} - T_{ext})\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T}. \quad (53)$$

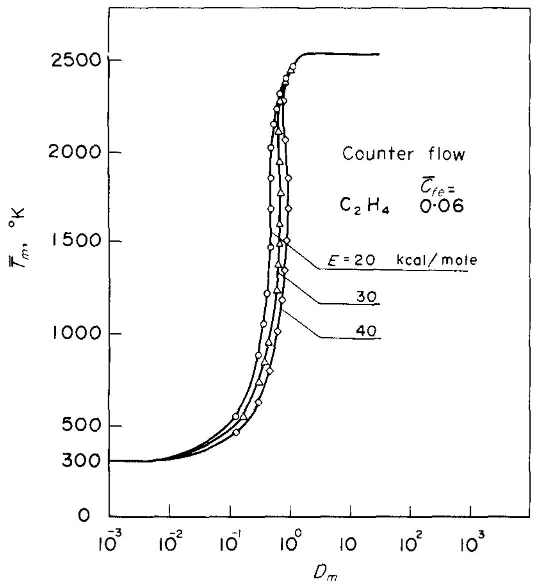

FIG. 8. Variation of T_m vs D_m for various activation energy for counter flow premixed flame.

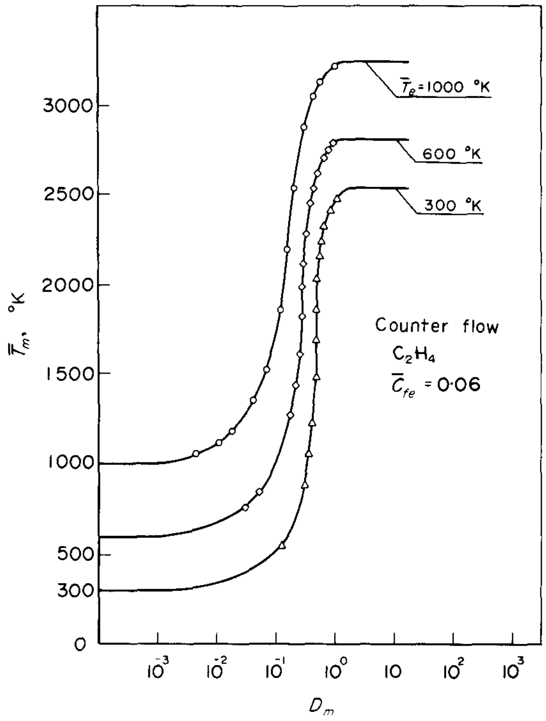

FIG. 10. Variation of T_m vs D_m for various free stream temperature for counter flow premixed flame.

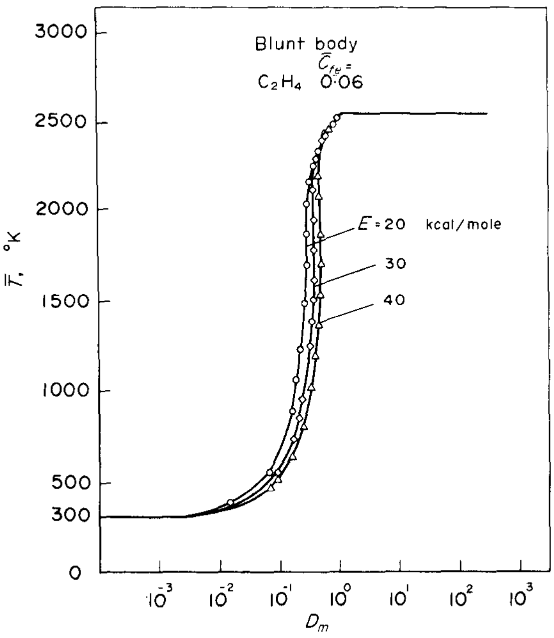

FIG. 9. Variation of T_m vs D_m for various activation energy for blunt body stagnation region premixed flame.

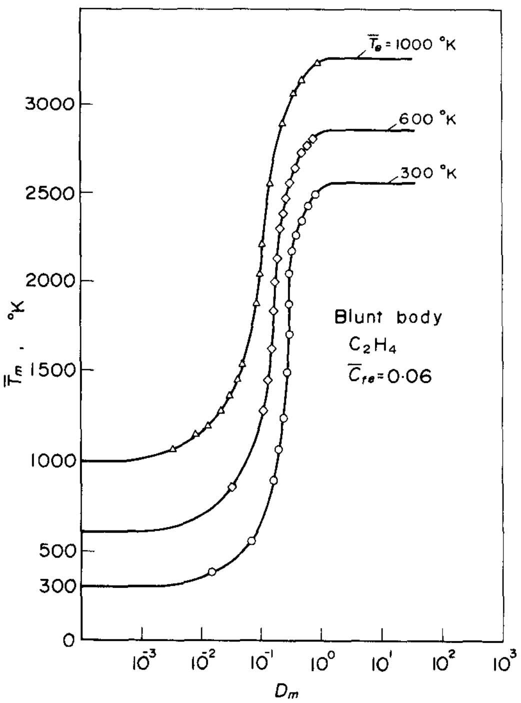

FIG. 11. Variation of T_m vs D_m for various free stream temperature for blunt body stagnation region premixed flame.

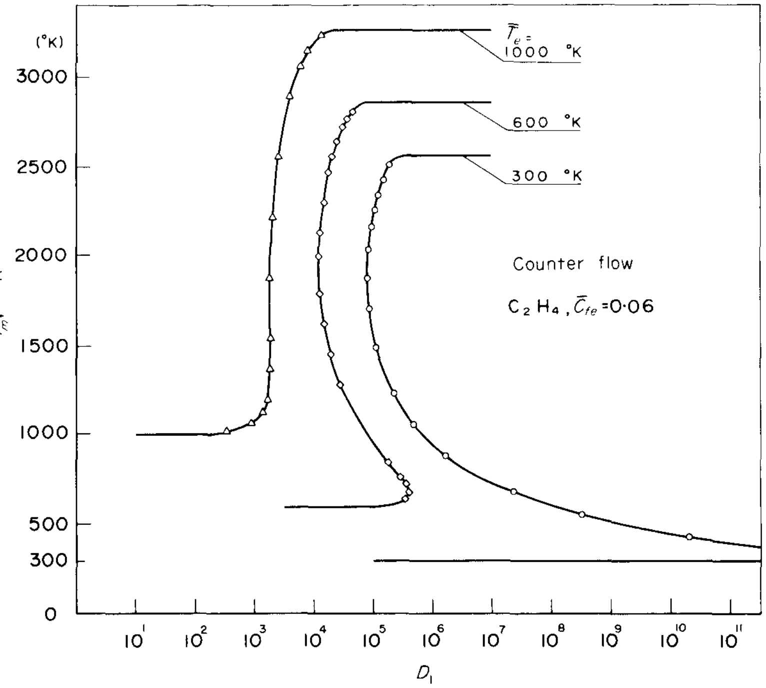

FIG. 12. Fendell curves showing the effect of free stream temperature for counter flow premixed flame.

(ii) in case of $T(0) < T_{ext}$

$$D_m = D_1[T_e + C_{fe} - T(0)] \times [T_e + C_{oe} - T(0)]\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T}. \quad (54)$$

4.4 *Factors which effect the maximum local Damköhler number at extinction*

In the previous section, the effect of varying fuel and oxidant concentrations was shown. In this section, the effects of varying activation energy, free stream temperature, specific heat at constant pressure, heat of combustion of fuel, kinds of fuel and Prandtl number, etc. are studied for both flow geometries, and the validity of the maximum local Damköhler number as an criterion for flame extinction was verified.

(a) *Effect of activation energy.* Effect of variation of activation energy is shown in Figs. 8 and 9 for counter flow and blunt body stagnation region flow, respectively. Variation of E has by far the less effect upon $D_{m,ext}$ than one upon $D_{1,ext}$.

(b) *Effect of free stream temperature.* Figures 10 and 11 show the effect of varying jet temperature for counter flow and blunt body stagnation flow, respectively. The T_m vs D_m curves are shifted to lower range of D_m with increasing jet temperature. No extinction state establishes when jet temperature is raised above a certain value, as was already noted by Chung *et al.* [17].

Fendell curves corresponding to Fig. 10 are also shown in Fig. 12, from which it can be seen that an aspect of abrupt change of maximum temperature at extinction in T_m vs D_m curves becomes vague and no extinction state is reached.

(c) *Effect of specific heat at constant pressure or heat of combustion of fuel.* As seen from equation (29), specific heat at constant pressure C_p and heat of combustion of fuel Δh_c appear only in a form of $C_p/\Delta h_c$, it is sufficient to examine the effect of either of two factors. Figures 13 and 14 show the effect of varying C_p for the two flow configurations. $D_{m,ext}$ remains fairly constant with variation of C_p, and therefore it is known that heat of combustion of fuel Δh_c also has little effect upon $D_{m,ext}$.

(d) *Effect of different kinds of fuel.* T_m–D_m curves are shown in Fig. 15 (counter flow) and in Fig. 16 (blunt body stagnation flow) for stoichiometric propane–air premixed flame as an example of different kinds of fuel. General features are the same as etylene–air flame.

(e) *Effect of Prandtl number.* The result of varying Prandtl number is plotted in Fig. 17 and in Fig. 18 for two flows.

Varying Prandtl number has almost no effect to T_m–D_m curves.

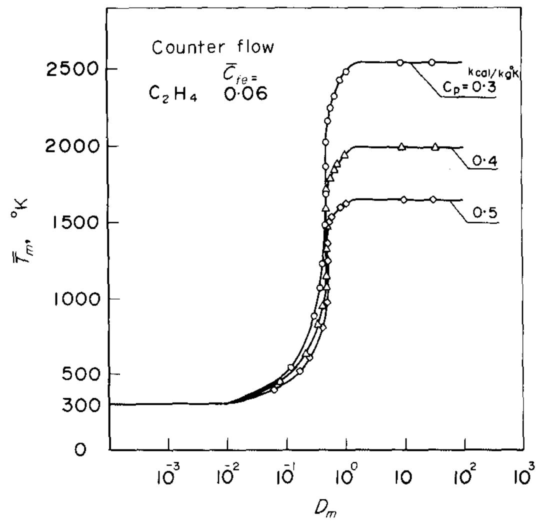

FIG. 13. Effect of variation of specific heat at constant pressure for counter flow premixed flame.

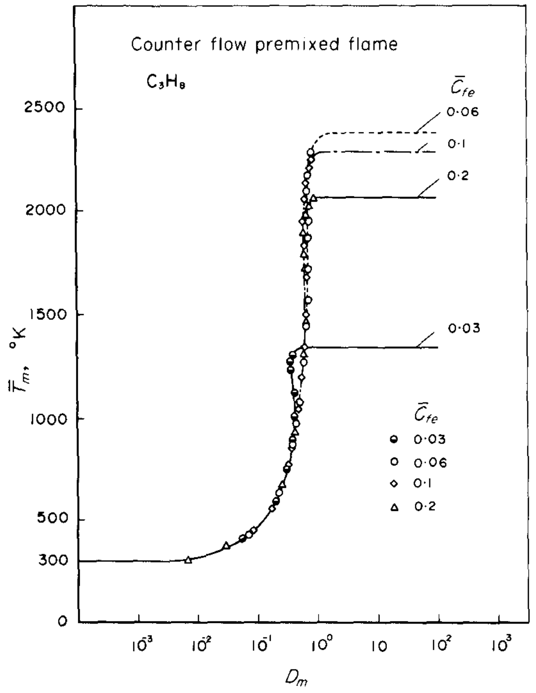

FIG. 15. T_m vs D_m curves of propane–air counter flow premixed flame.

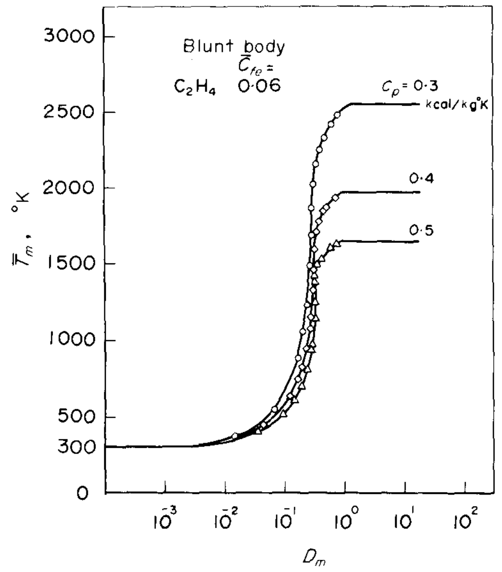

FIG. 14. Effect of variation of specific heat at constant pressure for blunt body stagnation region premixed flame.

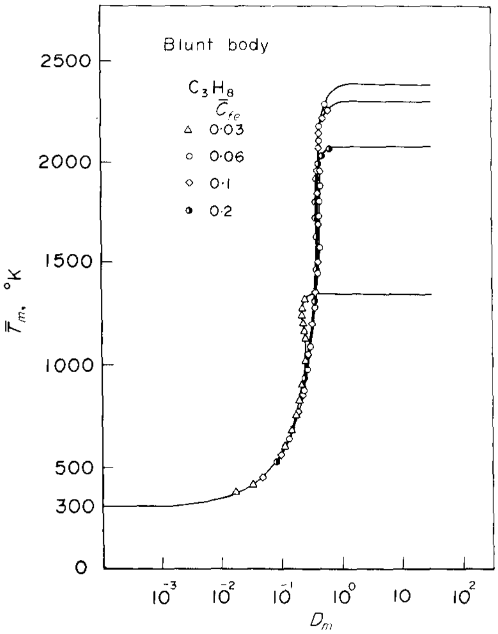

FIG. 16. T_m vs D_m curves of propane–air premixed flame for blunt body stagnation region flow.

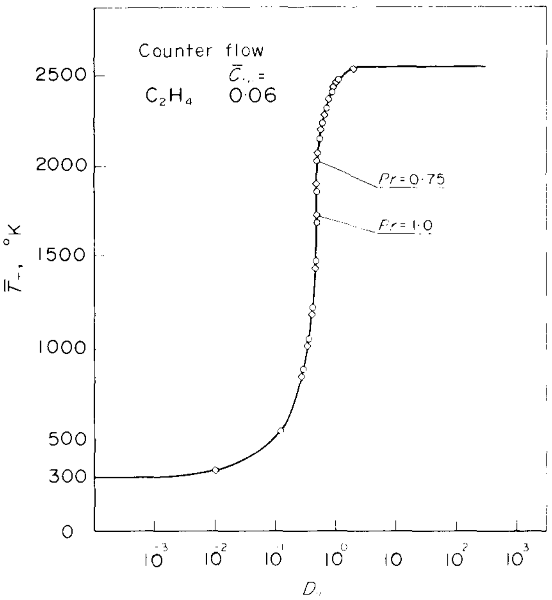

FIG. 17. Variation of T_m vs D_m for various Prandtl number (counter flow).

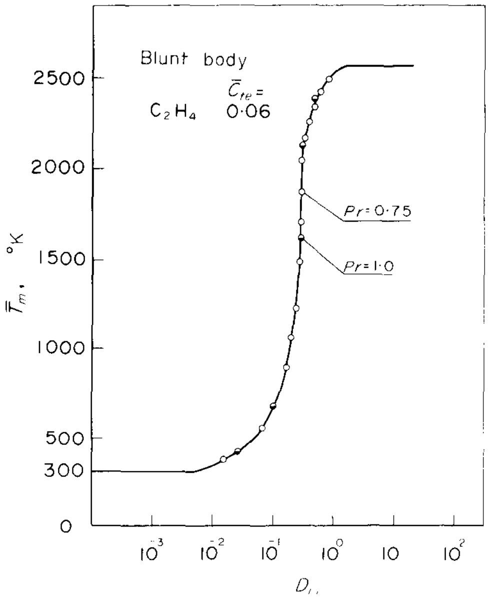

FIG. 18. Variation of T_m vs D_m for various Prandtl number (blunt body stagnation flow).

(f) *Effect of compressibility.* When the flow is assumed to be incompressible, the momentum equation can be decoupled from the energy equation, and the numerical calculations are considerably saved. Figures 19 and 20 show the effect of compressibility for two flows.

The compressibility contributes to shift the T_m–D_m curves to larger values of D_m.

4.5 *Extinction curves for constant velocity.* Taking account of the fact that flame extinguishes, as mentioned earlier, at nearly constant maximum local Damköhler number, the extinction curves (C_{fe}–$C_{\bar{o}e}$ curves) of constant velocity can be easily drawn without solving the ordinary differential equations. To obtain the $D_{m,\mathrm{ext}}/D_1$ constant curves, the following simultaneous equations should be solved numerically.

$$\frac{D_{m,\mathrm{ext}}}{D_1} = (T_e + C_{fe} - T)(T_e + C_{\bar{o}e} - T)\exp\left(-\frac{\theta}{T}\right)\frac{T_e}{T} \quad (55)$$

$$= C\ (\text{constant})$$

$$T^3 + \alpha T^2 + \beta T + \gamma = 0. \quad (56)$$

Here, α, β and γ are given by the equation (50) and C is a known constant.

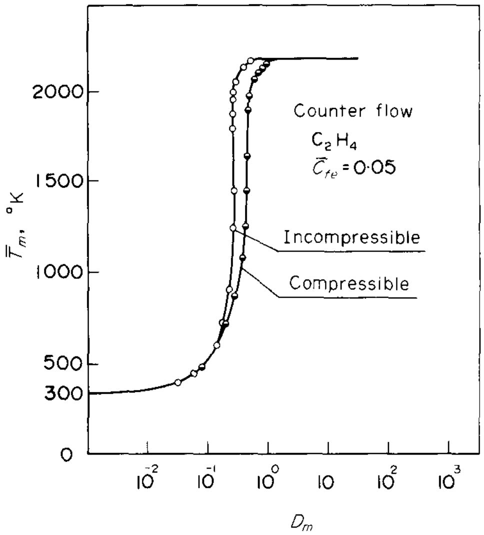

FIG. 19. Effect of compressibility for counter flow premixed flame.

Figure 21 shows the results of calculations for ethylene–air premixed flame. Region above solid curves designates the area where flame can exist. Since D_1 is inversely proportional to the velocity v_e, the extinction curve with small value of constant C implys the "weak flame", because the flames on this curve would extinguish at a lower flow velocity than the case with large C.

4.6 *Results for arbitrary Lewis number*

In the foregoing sections, the assumption of Lewis number unity was made for simplification of numerical works.

Most researches up-to-date have followed this assumption, except the one by Sharma and Sirignano [10]. They, however, used the properties evaluated at cold condition (300°K), which seems unrealistic because the temperature of flame zone is much higher than this

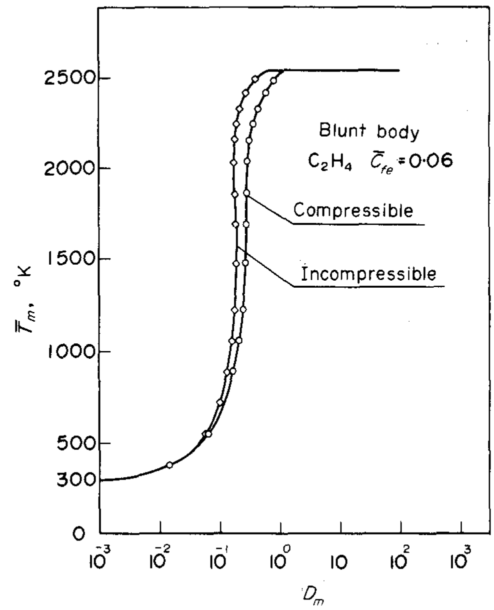

FIG. 20. Effect of compressibility for blunt body stagnation region premixed flame.

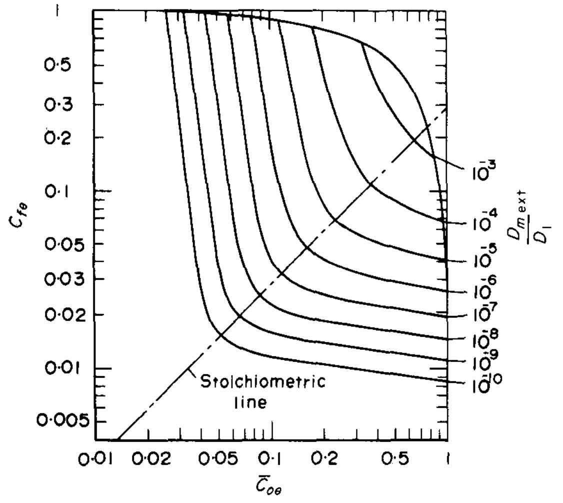

FIG. 21. Plot of C_{fe} vs C_{oe} at extinction for constant velocity. Dotted line shows stoichiometric mixture ratio.

temperature. Therefore, it is desirable to estimate the properties at some mean temperature.

As it is difficult to obtain the exact mean temperature for each condition, properties were calculated at extinction temperature $T_{ext} = 2000°K$ which was obtained for $Le_i = 1$, using evaluating formulae [18] for the burned gas of stoichiometric mixture of ethylene–air. Another properties evaluated at 1500°K were also used for comparison.

In numerical calculations, the results of Lewis number unity were used as first approximation. The results are shown in Figs. 22 and 23 for two flow geometries. As seen from these figures, variation of Lewis number has minor influence on T_m–D_m curves for both flows.

Fendell curves which correspond to Fig. 22 are also plotted in Fig. 24, from which it is known that nose points are moved toward higher flow velocity region.

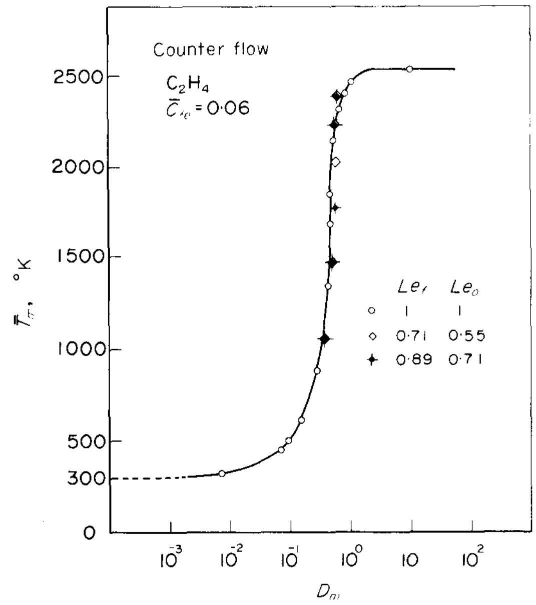

FIG. 22. Effect of variation of Lewis number for counter flow premixed flame.

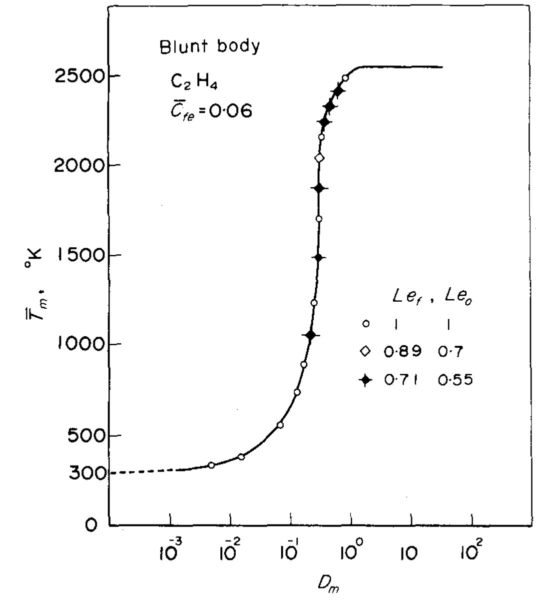

FIG. 23. Effect of variation of Lewis number for blunt body stagnation region premixed flame.

4.7 *Comparison of flow configurations*

(a) *Counter flow and blunt body stagnation flow.* Comparison of the results for counter flow with those for blunt body stagnation flow implys that the latter is "stronger flames" than the former, because $D_{m.ext}$ for the latter has smaller value than $D_{m.ext}$ for the former.

The cause of this difference may be explained by the concept of flame stretch.

Rate of flame stretch can be defined in this case by the velocity gradient $\partial u/\partial x$ at maximum local Damköhler number point. From equation (21), $\partial u/\partial x$ is expressed as

$$\frac{\partial u}{\partial x} = a\ f' = \frac{au}{u_e}, \tag{57}$$

which is the function of η alone. Comparing Fig. 3 with Fig. 4 leads that rate of flame stretch for blunt body stagnation flow is smaller than that for counter flow and then the above mentioned fact is derived.

(b) *Plane stagnation flow and axially symmetrical stagnation flow.* The difference between plane stagnation flow and axisymmetric stagnation flow is considered in the analysis through the parameters D_1 [see equation (37)] and η [see equation (26)].

It is immediately understood that the flame stabilized in plane stagnation flow is stronger than that in axisymmetric stagnation flow for the same coefficient "a" of potential equation $u_e = ax$.

5. CONCLUSIONS

The following conclusions may be drawn from the extinction analysis for premixed flames with counter flow and blunt body stagnation flow geometries.

(1) The use of non-dimensional parameter "maximum local Damköhler number" was proposed to predict flame extinction. Flames can be considered to extinguish at nearly constant values of this parameter.

(2) The effects of varying concentrations of fuel and oxidant, activation energy, free stream temperature, specific heat at constant pressure, kinds of fuel, Prandtl number, compressibility and Lewis number upon the maximum local Damköhler number were examined for both flow configurations.

(3) Comparing the results for two flows, it was pointed out that concept of flame stretch also holds for the present types of premixed flames. It can be explained that flames stabilized in blunt body forward stagnation flow have larger flame strength than that in counter flow.

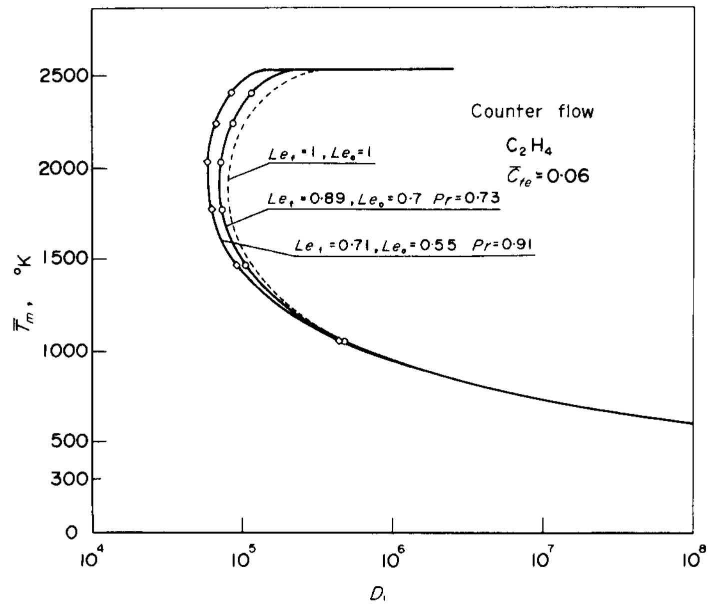

FIG. 24. Fendell curves for variation of Lewis number corresponding to Fig. 22 (counter flow premixed flame).

Acknowledgements—The author is thankful to Mr. Y. Eguchi for his help in numerical computations.

The author is also grateful to Computer Center at University of Tokyo for the use of HITAC 8800/8700 and Time Sharing Systems.

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ETUDE DE L'EXTINCTION D'UNE FLAMME PREMELANGEE DANS UN ECOULEMENT A CONTRE-COURANT ET REGION D'ARRET A L'AMONT D'UN OBSTACLE

Résumé – On étudie l'extinction des flammes prémélangées dans un écoulement à contre-courant et dans la région d'arrêt à l'amont d'un obstacle, à partir des approximations de la couche limite laminaire pour un écoulement bidimensionnel et axisymétrique de fluide visqueux et compressible. On adopte un nouveau paramètre adimensionnel de similitude, défini comme le nombre de Damköhler local maximal, qui est le critère d'extinction des flammes prémélangées. On considère pour différentes géométries les effets sur ce paramètre, des variations de concentrations de combustible et d'oxydant de la température de l'écoulement libre, de l'énergie d'activation, etc. . . . On conclue que l'"extinction" des flammes se produit pour une valeur presque constante du nombre maximal de Damköhler.

ANALYSE DES VERLÖSCHENS EINER VORGEMISCHTEN FLAMME BEI GEGENSTROM UND IM STAUBEREICH VOR EINEM STUMPFEN KÖRPER

Zusammenfassung – Über das Verlöschen einer vorgemischten Flamme bei Gegenstrom und im Staubereich vor einem stumpfen Körper wurde eine Untersuchung durchgeführt mit den Näherungsannahmen einer zweidimensionalen und axialsymmetrischen, viskosen, kompressiblen, laminaren Grenzschicht.

Ein neuer dimensionsloser Ähnlichkeitsparameter, die maximale örtliche Damköhler-Zahl, wurde als Kriterium für das Verlöschen vorgemischter Flammen angenommen. Die Einflüsse verschiedener Konzentrationen des Brennstoffs und des Sauerstoffträgers, der Freistromtemperatur, der Aktivierungsenergie, usw. auf diesen Ähnlichkeitsparameter wurden für beide Strömungsgeometrien untersucht. Aus der Analyse wurde geschlossen, daß "Verlöschen" von Flammen bei annähernd konstanten maximalen Damköhler-Zahlen eintritt.

АНАЛИЗ ЗАТУХАНИЯ ГОРЕНИЯ ПРЕДВАРИТЕЛЬНО ПЕРЕМЕШАННЫХ СМЕСЕЙ ВО ВСТРЕЧНОМ ПОТОКЕ В ПЕРЕДНЕЙ КРИТИЧЕСКОЙ ТОЧКЕ ПРИ ОБТЕКАНИИ ТУПОГО ТЕЛА

Аннотация — Анализируется затухание горения предварительно перемешанных смесей во встречных потоках в передней критической точке при обтекании тупого тела в приближении двухмерного и осесимметричного вязкого сжимаемого ламинарного пограничного слоя. В

качестве критерия затухания горения предварительно перемешанных смесей принимается новый безразмерный параметр подобия, определяемый как максимальное локальное число Дамкёлера.

Для встречных потоков исследовалось влияние изменения концентрации топлива и окислителя, температуры свободного потока, энергии активации и т. д. на параметр подобия. Из данного анализа следует, что «затухание» пламен имеет место при почти постоянных значениях максимального числа Дамкёлера.